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Optimal Workfare in Unemployment Insurance

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Nonparametric Engel Curves and Revealed Preference

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Abstract

This paper applies revealed preference theory to the nonparametric statistical analysis of consumer demand. It exploits the idea that price-taking individual households in the same market face the same relative prices, in order to smooth across the demands of individuals for each common price regime. This is shown to provide a stochastic structure within which to examine the consistency of household level data and revealed preference theory. An application is made to a long time series of repeated cross-sections from the 1974-1993 UK Family Expenditure Surveys. The consistency of this data with revealed preference theory is examined. Where rejections do occur, suitable adjustments to prices for quality or taste changes are explored. For periods of consistency with revealed preference bounds are placed on true cost of living indices.

Keywords: Consumer demands, nonparametric regression, revealed preference.

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1. Introduction[1]

The attraction of revealed preference theory is that it allows an assessment of the empirical validity of the usual integrability conditions without the need to impose particular functional forms on preferences. Although introduced by Afriat (1973) and Diewert (1973) to describe individual demands, it has usually been applied to aggregate data but this presents a number of problems[2]. First, on aggregate data, 'outward' movements of the budget line are often large enough, and relative price changes are typically small enough, that budget lines rarely cross (see Varian (1982), Bronars (1987) and Russell (1992)). This means that aggregate data may lack power to reject revealed preference (RP) conditions. Second, if we do reject RP conditions on aggregate data we have no way of assessing whether this is due to a failure at the micro level or to the inappropriate aggregation across households that do satisfy the integrability conditions but who have different non-homothetic preferences. Finally, it has proven difficult to devise tests of the significance of rejections in the yes/no context of RP tests. In this paper we develop and apply techniques which allow us to conduct a nonparametric analysis of micro data. By combining nonparametric statistical methods with a revealed preference analysis of micro data we can overcome the problems we have described.

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[2] See Manser and McDonald (1988), and references therein.

We also have a number of other motivations for this study. First, parametric demand studies on micro data often reject Slutsky symmetry which is one of the implications of utility maximisation subject to a linear budget constraint. Amongst the many possible explanations for this rejection are that either we have the 'wrong' functional form or that there exists no well-behaved form of preferences which can rationalise the data. Nonparametric analysis allows us to check this. Second, it has proven difficult to test for (global) negative semi-definiteness of the Slutsky matrix in parametric demand models. Using revealed preference nonparametric analysis we can simultaneously test for both symmetry and negative semi-definiteness. Third, if the integrability conditions are not rejected, we often wish to go on and use demand estimates for policy analysis. Using parametric analysis there is always some uncertainty as to how much the welfare conclusions are driven by functional form. If we employ nonparametric techniques then we can obtain bounds on welfare effects and use these bounds to judge the importance of the choice of functional form on welfare conclusions. Fourth, the nonparametric analysis can aid in the development of new and parsimonious parametric demand systems. Finally, we can extend the nonparametric analysis to investigate revealed preference for conditional demands.

We begin this paper by deriving a method for choosing a sequence of total expenditures that maximise the power of the test of the Generalised Axiom of Revealed Preference (GARP) with respect to a given preference ordering. This is shown to be implementable on micro data through the nonparametric estimation of local average demands, or local Engel curves, for each common price regime. We assume that households in each location and time period face the same relative prices. Under this assumption, the nonparametric Engel curves correspond to expansion paths for each price regime. These expansion paths are then shown to provide an attractive method for testing GARP on micro data. We also consider the use of conditional demands and separability in tests of GARP when preferences for a particular good, or group of goods, may be changing over time. For sequencies of relative prices where GARP is not rejected, the estimated nonparametric expansion paths are also shown to enable the construction of tight bounds on true cost of living indices and on the welfare effects of non-marginal relative price changes.

In estimation we address two key issues that arise when placing local average demands in a structural economic context. The nonparametric Engel curve for each commodity is specified as a kernel regression of the budget share on log total expenditure. First, we allow for the endogeneity of log total expenditure in the nonparametric budget share equations. We do this by completing the model with a reduced form specification for log total expenditure in terms of disposable income. The residual from this reduced form regression is added to the nonparametric Engel curve regression to control for the endogeneity of total expenditure. This augmented regression equation has a partially linear form and can be estimated using the semiparametric estimator suggested by Robinson (1988). Second, we consider the problem of pooling nonparametric Engel curves across households of different demographic compsition. We show that the shape invariant model of Härdle and Marron (1990) provides a theory consistent generalisation to the partially linear semiparametric method of pooling nonparametric Engel curves across household of different composition. A partially linear model for demographic variation is shown to reduce to Piglog demands under homogeneity and symmetry.

Using a long time series of repeated cross-sections we use these nonparametric regression curves, adjusted for endogeneity and demographic composition, to examine whether revealed preference theory can be rejected for particular types of individuals or in particular subperiods of the data. From the asymptotic distribution theory for nonparametric regression we are able to provide a statistical structure within which to examine the consistency of data with revealed preference theory without imposing a global parametric structure to preferences. The approach we adopt provides an alternative to the Afriat inefficiency measure explored in Famulari (1995) and Mattei (1994).

There remains the issue of unobserved heterogeneity. Even controlling for demographic composition taking two households that are similar in time, place and total expenditure we usually find that demand patterns are quite different. This makes the application of (RP) nonparametric techniques to micro data problematic. Even taking a small number of households in different price regimes usually leads to a rejection of the nonparametric conditions (see Koo (1963), Mossin (1972) and Mattei (1994), for example, and the recent paper by Sippel (1997) on the use of experimental data). We discuss conditions on preferences which are such that unobserved heterogeneity can be accommodated, and which also mean that our empirical approach successfully identifies average demand responses. We also investigate conditions under which average demand responses can usefully be used for measuring the welfare cost of non-marginal price changes and derive expressions for the resulting bias.

The layout of the paper is as follows. In Section 2 the method for choosing a sequence of total expenditures that maximise the power of the test of GARP with respect to a given preference ordering is developed. A framework for implementing this procedure by using nonparametric Engel curves is provided. This section also considers the use of conditional demands and develops a method of bounding true cost of living indices. Two algorithms are presented which give upper and lower bounds to a level set of utility passing through any point in commodity space chosen. Section 3 discusses preference heterogeneity and examines the relationship between the nonparametric Engel curves used to test GARP and the average demands (and average welfare) of a set of heterogeneous households upon which they are based. In Section 4 we discuss the data and present an empirical investigation of these ideas using twenty years of the British Family Expenditure Survey. Tight bounds for the true cost of living over this period are presented and shown to provide large improvements on classical revealed preference bounds. Important differences in the change in cost of living across income deciles are found. Section 5 concludes.

2. Individual Data and Revealed Preference

2.1. Revealed Preference and Observed Demands

Suppose we wished to test experimentally whether a particular agent had 'rational' and stable preferences. In the context of demand, this means facing them with a series of prices and total expenditures and testing whether their demand responses satisfy the Slutsky conditions. Specifically, if we have T time periods and given an n-vector of prices $\mathbf{p}_t$ in each period t we could present the agent with a series of total expenditures x_t and test whether the resulting time series of n-vector demands $\mathbf{q}_t = \mathbf{q}(\mathbf{p}_t, x_t) = \mathbf{q}_t(x_t)$ satisfy revealed preference tests. To do this, construct a $(T \times T)$ matrix m in which, for each pairwise comparison the (t,s) element defines an indicator variable:

$$m^{ts} = 1[\mathbf{p}_t'\mathbf{q}_t(x_t) \;\; \geq \;\; \mathbf{p}_t'\mathbf{q}_s(x^s)] \text{ for all } t,s = 1,...,T. \tag{2.1}$$

which is one when the revealed preference comparison in parentheses is satisfied (see Varian (1982)) and zero otherwise. We say that $\mathbf{q}_t(x_t)$ is *directly revealed weakly preferred* to $\mathbf{q}_s(x_s)$, $(\mathbf{q}_t(x_t)\ R^0\ \mathbf{q}_s(x_s))$ if the latter vector of quantities is affordable at period t prices and total expenditure. If the inequality in (2.1) is strict then we say that $\mathbf{q}_t(x_t)$ is *directly revealed strictly prefrerred* to $\mathbf{q}_s(x_s)$ $(\mathbf{q}_t(x_t)P^0\mathbf{q}_s(x_s))$ since the agent could have obtained the latter more cheaply (at the prices $\mathbf{p}_t$) but chose not to.

Now consider a sub-sequence of periods $\{s,t,u,...v,w\}$ where the order matters (so that the sub-sequence $\{s,t,u,...v,w\}$ differs from $\{t,s,u,...v,w\}$). We say that the sub-sequence of total expenditures $\{x_s,x_t,x_u,...x_v,x_w\}$ is *preference ordered* if $\{m^{st},m^{tu},...m^{vw}\} = \{1,1,...1\}$. Thus a sub-sequence of total expenditures is preference ordered if the demand associated with any total outlay is revealed at least as good as the next one. Given this, we define an *indirect* revealed preference relationship: $\mathbf{q}_s(x_s)$ is *indirectly revealed weakly preferred* to $\mathbf{q}_w(x_w)$ if there is a preference ordered sub-sequence starting in s and ending in w; we denote this by $\mathbf{q}_s(x_s)\ R\ \mathbf{q}_w(x_w)$. Given a matrix m of direct comparisons we can construct a matrix $\widetilde{m}$ of indirect comparisons by taking the transitive closure of m; Varian (1982) shows that this can be achieved inexpensively using Warshall's algorithm. Suppose now that we have a preference ordered sub-sequence $\{x_s,x_t,x_u,...x_v,x_w\}$ and that we also have that $\mathbf{q}_w(x_w)$ is directly revealed strictly preferred to $\mathbf{q}_s(x_s)$ so that:

$$\mathbf{p}_w'\mathbf{q}_w(x_w) > \mathbf{p}_w'\mathbf{q}_s(x_s) \tag{2.2}$$

In this case we say that this sub-sequence fails *GARP*, the general axiom of revealed preference. We shall say that the chronological sequence of total outlays $\{x_1, x_2, ...x_T\}$ fails GARP if some sub-sequence fails GARP.

Below, we shall also make use of the Afriat numbers. In terms of the Afriat inequalities (Varian (1982, p. 949), $\mathbf{q}_s \; R \; \mathbf{q}_w$ implies that there exist numbers $U_s, U_w, \lambda_w > 0$ such that

$$U_s \leq U_w + \lambda_w \mathbf{p}_w'(\mathbf{q}_s - \mathbf{q}_w). \tag{2.3}$$

If (2.2) holds then $\mathbf{p}_w'(\mathbf{q}_s - \mathbf{q}_w) < 0$, and since $\lambda_w > 0$ it must be that $U_s < U_w$ which is a failure of $\mathbf{q}_s \; R \; \mathbf{q}_w$ and consequently a failure of GARP.

2.2. Choosing a Path for Comparison Points

The choice of the sequence of total expenditures x_t used in the comparisons above requires some discussion. There is a well known problem with applying GARP tests to data in practice to which Varian (1982) refers in his applied work. This is to do with the fact that, particularly with annual data, income growth over time can swamp variations in relative prices (which are what we are interested in). This is because real income growth induces outward movements of the budget constraint and, combined with typically small period-to-period relative price movements, this means that budget lines may seldom cross. As a result, data often lacks power to reject GARP. Indeed, if we choose the x_t's so that budget lines never cross then we can never violate the GARP conditions. Clearly then, the power of a revealed preference test will depend critically on the choice of $(x_1, x_2, ...x_T)$.

One solution is to choose a sequence of constant "real" total expenditures.

Thus given x_1 and a set of price indices $(P_1(\mathbf{p}_1), P_2(\mathbf{p}_2), ...P_T(\mathbf{p}_T))$ we could choose $x_t = x_1 P_t / P_1$. Although superficially attractive this begs the question of what price index to use. More importantly, even if the series of demands generated in this way did satisfy GARP, we cannot be sure that any other series of total expenditures starting from x_1 would also satisfy GARP. Instead of this, we devise a simple algorithm for determining a sequence of x_t points through the data which maximises the chance of finding a rejection given a particular preference ordering of the data.

Suppose we have a sequence of demands and consider any preference ordered sub-sequence $\{x_s, x_t, x_u, ...x_v, x_w\}$[3]. The algorithm for choosing the most powerful path for this preference ordered sub-sequence is a recursive scheme. Given total expenditure in the last period in the sub-sequence, x_w, total outlay in the second to last period v is chosen so that the period w bundle is just affordable at the period v prices; denote this $\tilde{x}_v = \mathbf{p}_v' \mathbf{q}_w(x_w)$. Thus $\mathbf{q}_v(\tilde{x}_v)$ is directly revealed weakly preferred to $\mathbf{q}_w(x_w)$. Then total outlay in the previous period is chosen so that $\mathbf{q}_v(\tilde{x}_v)$ is just affordable and so on. Thus the *sequential maximum power* (SMP) path for the preference ordered sub-sequence $\{x_s, x_t, x_u, ...x_v, x_w\}$ is given by:

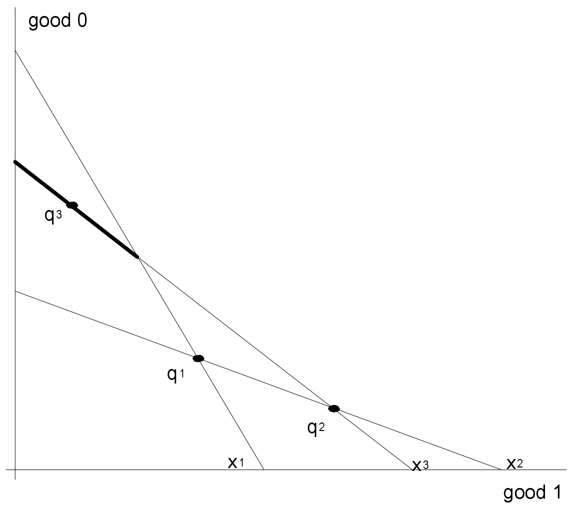
$$\{\tilde{x}_s, \tilde{x}_t, \tilde{x}_u, ...\tilde{x}_v, x_w\} = \{\mathbf{p}_s' \mathbf{q}_t(\tilde{x}_t), \mathbf{p}_t' \mathbf{q}_u(\tilde{x}_u), ...\mathbf{p}_v' \mathbf{q}_w(\tilde{x}_w), x_w\} \tag{2.4}$$

By construction, any SMP path is preference ordered. Figure 2.1 illustrates a three period, two good example in which the order of the sub-sequence is $\{\mathbf{q}_3 R^0 \mathbf{q}_2 R^0 \mathbf{q}_1\}$. In figure 2.1 the shaded part of the period 3 budget line gives the $\mathbf{q}_3$ demand points

[3] We are only interested in sequences over the whole data period in which there is some possibility of rejecting GARP. This implies that there must be at least one preference ordered sub-sequence.

which result in a rejection of GARP. Intuitively, one can see that this path is going to maximise the probability of finding some rejection in the sense that 'pushing out' either of the period 2 or 3 budget lines will reduce the length of the rejection region. If demands are normal then this reduces the chance of observing a demand in that region.

Figure 2.1: Testing GARP, a three period, two good example



To formalise our notion of power we need some more definitions. The first concern demands. Let $q_t^i(x)$ be the ith element of the demand vector in period t. Demands are said to be *normal* if $x > x'$ implies that $q_t^i(x) > q_t^i(x')$ for all (i, t). This is a natural assumption to make for the wide commodity aggregates we shall be working with. In the data analysis below we show that the 22 commodities we deal with are all normal goods. We shall also need: demands are *continuous* if $q_t^i(x)$ is a continuous function for all (i, t).

The second definition is required to state our formal result. A *cycle* of a sub-sequence of indices is defined to be another sub-sequence of the same length that starts at any point in the original sub-sequence and maintains the ordering of indices; for example $\{u, ...v, w, s, t\}$ is a cycle of the sub-sequence $\{s, t, u, ...v, w\}$ (and *vice versa*). Similarly we say that $\{x_u, ...x_v, x_w, x_s, x_t\}$ is a cycle of $\{x_s, x_t, x_u, ...x_v, x_w\}$.

Proposition 1. *Suppose that the budget sequence $(\hat{x}_1, \hat{x}_2, ...\hat{x}_T)$ has a preference ordered sub-sequence that rejects GARP. If demands are normal then the SMP path for any utility non-decreasing cycle of that sub-sequence also rejects GARP.*

Proof. Without loss of generality we take the length of the GARP rejecting preference ordered sub-sequence to be 3; let it be, say, $\{\hat{x}_s, \hat{x}_t, \hat{x}_u\}$. That is:
(1) $\hat{x}_s = \mathbf{p}_s'\mathbf{q}_s(\hat{x}_s) \geq \mathbf{p}_s'\mathbf{q}_t(\hat{x}_t)$ and
(2) $\hat{x}_t = \mathbf{p}_t'\mathbf{q}_t(\hat{x}_t) \geq \mathbf{p}_t'\mathbf{q}_u(\hat{x}_u)$ and
(3) $\hat{x}_u = \mathbf{p}_u'\mathbf{q}_u(\hat{x}_u) > \mathbf{p}_u'\mathbf{q}_s(\hat{x}_s)$.
We consider the SMP path for one cycle of this ordered sub-sequence and show that it too rejects GARP; the proof for any other cycle follows similar lines. Consider the cycle $(\hat{x}_t, \hat{x}_u, \hat{x}_s)$. The corresponding SMP path $(\tilde{x}_t, \tilde{x}_u, \hat{x}_s)$ has:
(4) $\tilde{x}_u = \mathbf{p}_u'\mathbf{q}_s(\hat{x}_s) = \mathbf{p}_u'\mathbf{q}_u(\tilde{x}_u)$ and
(5) $\tilde{x}_t = \mathbf{p}_t'\mathbf{q}_u(\tilde{x}_u) = \mathbf{p}_t'\mathbf{q}_t(\tilde{x}_t)$.
By construction this is a preference ordered sub-sequence ($\mathbf{q}_u(\tilde{x}_u)R^0\mathbf{q}_s(\hat{x}_s)$ and $\mathbf{q}_t(\tilde{x}_t)R^0\mathbf{q}_u(\tilde{x}_u)$) so that this sub-sequence rejects GARP if $\mathbf{q}_s(\hat{x}_s)P^0\mathbf{q}_t(\tilde{x}_t)$; that is, if:
(6)$\mathbf{p}_s'\mathbf{q}_s(\hat{x}_s) > \mathbf{p}_s'\mathbf{q}_t(\tilde{x}_t)$.
From normality, (3) and (4) we have:
$\mathbf{p}_u'\mathbf{q}_u(\hat{x}_u) > \mathbf{p}_u'\mathbf{q}_u(\tilde{x}_u) \Rightarrow \hat{x}_u > \tilde{x}_u \Rightarrow q_u^i(\hat{x}_u) > q_u^i(\tilde{x}_u)$ for all i
Combining this with (2) and (5) we have:
$\mathbf{p}_t'\mathbf{q}_t(\hat{x}_t) \geq \mathbf{p}_t'\mathbf{q}_u(\hat{x}_u) > \mathbf{p}_t'\mathbf{q}_u(\tilde{x}_u) = \mathbf{p}_t'\mathbf{q}_t(\tilde{x}_t) \Rightarrow \hat{x}_t > \tilde{x}_t$
From this and (1) we have:
$\mathbf{p}_s'\mathbf{q}_s(\hat{x}_s) \geq \mathbf{p}_s'\mathbf{q}_t(\hat{x}_t) > \mathbf{p}_s'\mathbf{q}_t(\tilde{x}_t)$
which is condition (6); hence GARP is rejected for this sub-sequence. ■

Thus if we test for GARP along a given SMP path starting from a given total expenditure and we do not reject, then we can be confident that we would not reject for any other path which starts from the same total expenditure and maintains the utility non-decreasing ordering implied by the SMP path. There is no need for the chosen ordering of the SMP path to be chronological.

In the statement of this proposition the normality assumption is necessary in the sense that without it we can construct counter-examples. In the data analysis below we show that the 22 commodities we deal with are all normal goods.

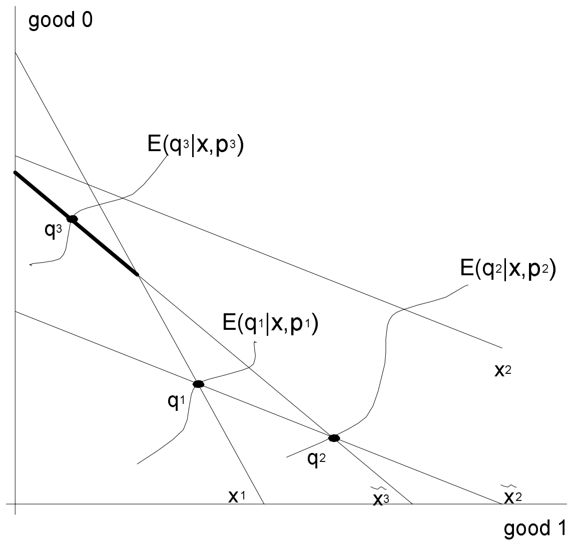
2.3. Nonparametric Expansion Paths

So far we have been assuming that we can take a single agent and present them with any path of total expenditures. In practice, of course, we cannot do this in anything but an experimental setting. Instead we have to use non-experimentally generated data on prices and quantities from a number of heterogeneous households observed only once.

To achieve the SMP path described in Proposition 1, we need to be able to move individuals along their expansion paths. To see this consider Figure 2.2 which adds expansion paths to Figure 2.1. Suppose we start at budget line x_1 for prices $\mathbf{p}_1$ and demands are given by $\mathbf{q}_1(x_1)$. To follow the SMP path we need to choose a budget line under prices $\mathbf{p}_2$ so that $\mathbf{q}_1$ can just be bought, this budget level at prices $\mathbf{p}_2$ is labelled $\widetilde{x}_2$. Then the chosen $\mathbf{q}_2(\widetilde{x}_2)$ is such that $\mathbf{q}_2(\tilde{x}_2)\ R^0\ \mathbf{q}_1(\hat{x}_1)$. If the observed budget was x_2 this is an example in which the budget line has moved out, reducing the chance of finding a rejection.

The sequential maximum power path can be constructed by moving along the $\mathbf{q}_2$ expansion path $E(\mathbf{q}_2|\ x, \mathbf{p}_2)$ in Figure 2.2. If we now choose $\widetilde{x}_3$ so that $\mathbf{q}_2$ can

Figure 2.2: Testing GARP, a three period, two good example with expansion paths



just be bought at $\mathbf{p}_3$ then we maximise the region of rejection of $\mathbf{q}_3$ with respect to $\mathbf{q}_2$, denoted by the thick part of the budget line defined by $\mathbf{p}_3'\mathbf{q}_3$. At any point in this region $\mathbf{q}_1(\tilde{x}_2)\ P^0\ \mathbf{q}_3(\hat{x}_3)$ and GARP is rejected. Again this can only be achieved by moving along the $\mathbf{q}_3$ expansion path to the point $\mathbf{q}_3$ on budget line $\widetilde{x}_3$.[4] Movements along the expansion path are equivalent to movements along the Engel curve. So that if Engel curves are known then so are the expansion paths. To estimate the Engel curves we turn to nonparametric regression methods.

It is assumed that in period t, prices denoted $\mathbf{p}_t^j$ for each good $j = 1, ..., n$ are common to all individuals. For our purposes, it will be useful to think of t as time but it may alternatively reflect region or some other separation within which the same market price is set. Typically the number of different price regimes $t = 1, ...T$, will be small. Commodity demands and total expenditure, on the other hand, are indexed by both an individual index i and t, the dimension of i will be (very) large.

The advantage of micro demand data is that we can estimate Engel curves nonparametrically for each common price regime. At any point in time and at any location all individuals face the same relative prices and are characterised by differences in endowment or total budget x_{it}. For each individual i and good j there is an expenditure $p_t^j q_t^{ji}$ in period t. In the next sub-section we discuss allowing for heterogeneity in preferences; for now we simply define the nonparametric Engel curve for price regime $\mathbf{p}_t$ as the mean expenditure *conditional* on total

[4] Note that the possibility of rejection is only maximised between adjacent points in the sequence, i.e. between the demands under price regimes 1 and 2, 2 and 3 etc. In this case the rejection we find is between $\mathbf{q}_1$ and $\mathbf{q}_3$. To maximise the possibility of finding *this* rejection we could choose the SMP comparison such that $\widetilde{x}_3$ is selected such that $\mathbf{q}_3(\tilde{x}_3)\ R^0\ \mathbf{q}_1(\hat{x}_1)$, i.e. choose the price/ preference ordering sequence **1, 3, 2**.

outlay x i.e.

$$E(p_t^j q_t^j | x) = p_t^j g_t^j(x). \tag{2.5}$$

The price p_t^j is a constant in each period t so that, in any price regime $\mathbf{p}_t$, the conditional mean of each demand given total outlay x defines a set of cross-section demands:

$$E(q_t^j | x) = g_t^j(x) \text{ for } j = 1, ..., n. \tag{2.6}$$

The power of the nonparametric analysis comes from knowledge of the regression line $g_t^j(x)$ and its precision local to specific points of the x distribution.

From extensive earlier work on the Engel curve relationship in British household level data (see Banks, Blundell and Lewbel (1997)), we know that budget shares that are linear in log total expenditure provide a good baseline specification. For this reason we estimate the Engel curves using the nonparametric regression of budget shares on log total outlay.[5] Defining budget shares as

$$w_t^j \equiv \frac{p_t^j q_t^j(x)}{x} \text{ for } j = 1, ..., n, \text{and } t = 1, ..., T, \tag{2.7}$$

the nonparametric regression estimates the conditional expectation

$$E(w_t^j | x) = m_t^j(\ln x) \text{ for } j = 1, ..., n, \text{ and } t = 1, ..., T. \tag{2.8}$$

In what follows we will refer to $m_t^j(\ln x)$ as the local average demand for good j in period t indexed by x.

2.4. Pointwise Inference for Pairwise Comparisons

At each stage in the above discussion we are comparing weighted sums of kernel regressions. The pairwise comparison in (2.2) can be written

[5] Banks, Blundell and Lewbel (1997) find the density of $\ln x$ to be well approximated by a normal density in their Engel curve study of the same UK Family Expenditure Data source.

$$\sum_{j=1}^{n} p_t^j g_t^j(x_t) > \sum_{j=1}^{n} p_t^j g_s^j(x_s) \text{ for } s \neq t. \tag{2.9}$$

Noting that adding-up implies

$$\sum_{j=1}^{n} p_t^j g_t^j(x_t) \equiv x_t \text{ for all } t$$

condition (2.9) conveniently reduces to the comparison

$$\sum_{j=1}^{n-1} \alpha_{ts}^j g_s^j(x_s) < x_t - \delta_{ts} x_s, \tag{2.10}$$

where $\alpha_{ts}^j = p_t^j - \frac{p_t^n}{p_s^n} p_s^j$ and $\delta_{ts} = \frac{p_t^n}{p_s^n}$ are known constant weights in each price regime.

Since the nonparametric Engel curve has a pointwise asymptotic standard error we can evaluate the distribution of each $g_t^j(x_t)$ at a finite set of points x_t. For example, in what follows we consider certain quantile points on the SMP path. Pointwise standard errors for kernel regression are given in Härdle (1990).[6]

[6] Briefly, for bandwidth choice h and sample size N the variance can be well approximated at point x for large samples by

$$var(g^j(x)) \simeq \frac{\sigma_j^2(x) c_K}{N h f_h(x)} \tag{2.11}$$

where c_K is a known constant and $f_h(x)$ is an (estimate) of the density of x

$$\sigma_j^2(x) = N^{-1} \sum_{i=1}^{N} \omega_h^i(x)(q^{ji} - g^j(x))^2$$

with weights from the kernel function

$$\omega_h^i(x) = K_h(x - x_t^i)/f_h(x).$$

To evaluate (2.10) we need to find the distribution of the weighted sum of correlated kernel regression estimates. However, since the g^j kernel estimates are to be evaluated at the same point x using the same kernel smoother and the same bandwidth, the constants associated with the kernel function and the density $f_h(x)$ itself will be common to all variance and covariance terms. Pointwise standard errors and confidence bands for expression (2.10) are therefore tractable and are used extensively in the empirical application below.

2.5. Quality Change, Conditional Demands, Separability and GARP

It is common in empirical demand analysis to work with conditional demands (see for example Browning and Meghir (1991)). This is particularly convenient where some good, or group of goods, is considered to be rationed or subject to some unmeasured change in quality, preference or habit formation, and is also not separable from the group of goods under study. For example, demands for tobacco consumption are very likely to be subject to changes in preference and quality following government health announcements over the period of study. It is unlikely that the level and participation of tobacco consumption is therefore fully rationalisable by a set of stable preferences over this period. However, it is also likely that preferences over certain other goods of interest, such as beer, wine, spirits and entertainment are directly affected by tobacco consumption; that is, they do not form a subgroup which is separable from tobacco. Consequently, demands conditional on the level of tobacco consumption may be rationalised even though for the set of goods with tobacco included this would not be the case. Similarly, the set of goods excluding tobacco would also not be rationalised in the case where they were not separable from tobacco consumption.

If there is an argument that preferences for tobacco may have changed over the period, then there is good reason to expect that a dataset which includes tobacco will fail a test of GARP. If this is so, then it means that separability is formally, as well as intuitively, rejected and we cannot simply omit tobacco from the set of goods considered.[7]

Consider instead the case of $n+1$ goods in which the 'conditioning' good q^0 is subject to some ration or quality change and preferences over the remaining 'goods of interest' $q^1, ..., q^n$ are thought to behave according to rational consumer theory[8]. Note that if preferences over the goods of interest are assumed not to be separable from the conditioning good, and we do not observe the latter, then we can rationalise any set of prices and quantities for the goods of interest (see Varian (1986)). Thus a 'missing' good makes it impossible to test for GARP.

The simple choice model is as follows

$$\max U\left(\mathbf{q}_t\right)$$

[7] To see why this is so consider a dataset which is partitioned into two sub-sets of goods and prices, $((\mathbf{p}^k, \mathbf{q}^k), (\mathbf{p}^0, \mathbf{q}^0))$. Preferences over $\mathbf{q}^0$ are weakly separable from $\mathbf{q}^k$ if there exists a sub-utility function $w(\cdot)$ and a super-utility function $v\left(\mathbf{q}^k, w\right)$ which is strictly increasing in w such that

$$u\left(\mathbf{q}^k, \mathbf{q}^0\right) \equiv v\left(\mathbf{q}^k, w\left(\mathbf{q}^0\right)\right)$$

The criterion for separability is set out by Varian (1983). This is that if the data were generated by such a utility function, then the data $(\left(\mathbf{p}^k, \mathbf{p}^0\right), \left(\mathbf{q}^k, \mathbf{q}^0\right))$ and the data $\left(\mathbf{q}^0, \mathbf{p}^0\right)$ must satisfy GARP. This is necessary. For sufficiency the data $\left(\mathbf{p}^0, \mathbf{q}^0\right)$ and $\left(\mathbf{p}^k, 1/\mu; \mathbf{q}^k, w\right)$ must satisfy GARP for some choice of (w, μ) which satisfy Afriat inequalities. In other words, the whole dataset has to pass GARP, the arguments of the sub-utility function have to pass GARP, and the whole dataset with the separable components replaced by their group 'price' ($1/\mu$ where μ is the marginal utility of income at $\mathbf{p}^{0\prime}\mathbf{q}^0$) and their group 'quantity' (w) must pass GARP.

[8] This idea of introducing a conditioning good is similar to the ideas proposed by Prais and Houthakker (1955) and Fisher and Shell (1971) and generalised by Muellbauer (1975). They introduce a time-varying quality parameter directly into the utility function.

$$\text{s.t. } \boldsymbol{\pi}_t'\mathbf{q}_t \leq x_t \quad t = 0, ..., T$$

where $\boldsymbol{\pi}_t = \mathbf{p}_t \cdot \boldsymbol{\mu}_t$ is the price which can be decomposed into the observed component $\mathbf{p}_t$ and an adjustment factor $\boldsymbol{\mu}_t$ (in the Prais Houthakker model is is a quality deflator). We assume that only one good (good 0) is subject to perference variation. Hence $\pi_t^k = p_t^k$ for $k \neq 0$, $\forall\ t$, and $p_0^0 = \pi_0^0$ (normalising μ_0^0 to 1), with $p_t^0 \gtrless \pi_t^0$ depending on whether the marginal utility of the good is is increasing or decreasing over time. If the marginal utility is increasing then $p_t^0 > \pi_t^0$ as $\mu_t^0 < 1$.

The first order condition is that

$$U'(\mathbf{q}_t) - \lambda_t (\mathbf{p}_t \cdot \boldsymbol{\mu}_t) = 0$$

where $\lambda_t > 0$ if the constraint binds. Substituting into the usual concavity conditions we have the Afriat inequality:

$$U_s \leq U_t + \lambda_t (\mathbf{p}_t \cdot \boldsymbol{\mu}_t)' (\mathbf{q}_s - \mathbf{q}_t) \tag{2.12}$$

or more specifically (denoting $\mathbf{q}_t \equiv (q_t^1, \ldots, q_t^n)$ and $\boldsymbol{\pi}_t \equiv (\pi_t^1, \ldots, \pi_t^n)$)

$$\begin{aligned} U_s &\leq U_t + \lambda_t \boldsymbol{\pi}_t' (\mathbf{q}_s - \mathbf{q}_t) && \text{if } t = 0 \\ U_s &\leq U_t + \lambda_t \boldsymbol{\pi}_t' (\mathbf{q}_s - \mathbf{q}_t) + \lambda_t \pi_t^0 (q_s^0 - q_t^0) && \text{otherwise.} \end{aligned}$$

We observe the correct prices for all the other goods ($k \neq 0$), $\forall\ t$ (since there is assumed to be no quality or perference variation). However, we do not observe the price for the 0th good (π_t^0), except in the 0th period.

The restrictions imposed by GARP in this case can be shown using a variant of Theorem 7 in Varian (1983) to imply a set of concavity conditions for the maximisation of some continuous, concave, monotonic and non-satiated utility function defined over $\mathbf{q} = (q^1, ..., q^n)'$ conditional on q^0.

Proposition 2. *The data* $(q_1^0, q_2^0, ...q_T^0, \mathbf{q}_1, \mathbf{q}_2, ...\mathbf{q}_T, p_1^0, p_2^0, ...p_T^0, \mathbf{p}_1, \mathbf{p}_2, ...\mathbf{p}_T)$ *can be rationalised iff there exist numbers* $U_s, U_t, \lambda_s > 0$ *and* μ_s *such that*

$$U_t \leq U_s + \lambda_s \mathbf{p}_s'(\mathbf{q}_t - \mathbf{q}_s) + \lambda_s p_s^0 \mu_s (q_t^0 - q_s^0) \tag{2.13}$$

Proof.

The proof is identical to that of Theorem 7 in Varian (1982).

■

Since the preference change (or quality change) model can be rewritten as a stable preference model with virtual prices the usual GARP restrictions apply to the data with the actual price of the replaced by the adjusted price (π_t^0). Thus allowing for a conditioning good is as though we can choose a price for this good that is different from the observed market price. If we can find μ_s's for each period that equal unity then we can rationalise the data on all $n+1$ goods. But if GARP is rejected for the full set of goods, the addition of the extra free variables μ_s may make it possible to rationalise the conditional demands for the goods of interest. Formally, $\mu_s p_s^0$ is the virtual price for the conditioning good in period s. If agents like the conditioning good less over time then we would expect to find that $\mu_t > \mu_s$ for $t > s$; that is, it is as though the *virtual* price of the conditioning good is rising over time. Adding more conditioning goods further relaxes the restrictions GARP places on the observed data.

In general, for some rejection such as $\mathbf{p}_t'\mathbf{q}_t \geq \mathbf{p}_t'\mathbf{q}_s$ and $\mathbf{p}_s'\mathbf{q}_s > \mathbf{p}_s'\mathbf{q}_t$ the minimum price adjustment to the price of the $0th$ good necessary such that $\mathbf{p}_s'\mathbf{q}_s = \mathbf{p}_s'\mathbf{q}_t$ is to set

$$p_s^0 = \frac{\sum_{k=1}^{K} p_s^k \left(q_s^k - q_t^k\right)}{\left(q_t^0 - q_s^0\right)} \tag{2.14}$$

If $(q_s^0 - q_t^0) < 0$, then this is a lower bound on p_s^0 (denoted by $\underline{p}_s^0$); i.e., we need to set $p_s^0 \geq \underline{p}_s^0$ so that $\mathbf{p}_s'\mathbf{q}_s \leq \mathbf{p}_s'\mathbf{q}_t$. Any $p_s^0 < \underline{p}_s^0$ will violate GARP. Reversing these inequalities this is an upper bound on p_s^0 (denoted by $\overline{p}_s^0$)[9]

Proposition 3. *If* $(q_t^0 - q_s^0) > 0$ *and* $\sum_{k=1}^{K} p_s^k \left(q_t^k - q_s^k\right) > 0$ *then set* $p_s^0 = \overline{p}_s^0$. *Any* $p_s^0 > \overline{p}_s^0$ *will violate GARP.*

Proof.

(1) Denote $\overline{\mathbf{p}}_s = \left(\overline{p}_s^0, p_s^1, ..., p_s^K\right)$

(2) $\overline{\mathbf{p}}_s$ is such that $\overline{\mathbf{p}}_s'\mathbf{q}_s = \overline{\mathbf{p}}_s'\mathbf{q}_t = x_s$

(3) Suppose $\underline{\mathbf{p}}_s > \overline{\mathbf{p}}_s$ where $\underline{\mathbf{p}}_s = \left(\underline{p}_s^0, p_s^1, ..., p_s^K\right)$.

(4) Then from (2) and (3) $\underline{\mathbf{p}}_s'\mathbf{q}_s > \overline{\mathbf{p}}_s'\mathbf{q}_s = \underline{\mathbf{p}}_s'\mathbf{q}_t \Rightarrow \mathbf{q}_s\ P\ \mathbf{q}_t$, but the SMP path induces that $\mathbf{q}_t R^0 \mathbf{q}_s$ which is a violation of GARP. ■

Now suppose that there are two rejections: $\mathbf{p}_r'\mathbf{q}_r > \mathbf{p}_r'\mathbf{q}_s \Rightarrow \mathbf{q}_r\ P^0\ \mathbf{q}_s$ and $\mathbf{p}_r'\mathbf{q}_r > \mathbf{p}_r'\mathbf{q}_t \Rightarrow \mathbf{q}_r\ P^0\ \mathbf{q}_t$, while the SMP path induces that $\mathbf{q}_t\ R^0\ \mathbf{q}_s$, $\mathbf{q}_s\ R^0\ \mathbf{q}_r$ and $\mathbf{q}_t\ R\ \mathbf{q}_r$. We need to find a single p_r^0 such that $\mathbf{p}_r'\mathbf{q}_r = \mathbf{p}_r'\mathbf{q}_s$ and $\mathbf{p}_r'\mathbf{q}_r = \mathbf{p}_r'\mathbf{q}_t$ (which is the minimum adjustment necessary). If $(q_s^0 - q_r^0) > 0$ and $(q_t^0 - q_r^0) > 0$ then we have two lower bounds of which the highest, $\max(\underline{p}_{rs}^0, \underline{p}_{rt}^0)$, encompasses the other and is the overall lower limit. Similarly if we have two upper limits then $\min(\overline{p}_{rs}^0, \overline{p}_{rt}^0)$ encompasses the other. But, if $(q_s^0 - q_r^0) > 0$ and $(q_t^0 - q_r^0) < 0$, say, then the first equation gives a lower limit for $p_r^0 \geq \underline{p}_r^0$, and the second gives an

[9]Note that $\sum_{k=1}^{K} p_s^k \left(q_s^k - q_t^k\right)$ and $\left(q_t^0 - q_s^0\right)$ may have different signs. In this case the minimum necessary adjustment will give a negative price. If this is an upper limit then no positive price for this good can be found which can rationalise GARP.

upper limit of $p_r^0 \leq \overline{p}_r^0$. If $\overline{p}_r^0 > \underline{p}_r^0$ then no value for p_r^0 in the interval will cause a violation of GARP. If $\overline{p}_r^0 < \underline{p}_r^0$ then there exists no value for p_r^0 which does not violate GARP[10].

2.6. Welfare bounds and expansion paths

Afriat (1977) described the way in which revealed preference information can be used to improve the classical bounds of the welfare effects of a price change. The idea is that the axioms of revealed preference can be used to glean additional information on the curvature of indifference surfaces in commodity space and that this can be used to improve classical two sided bounds on the welfare effects of price changes. This technique is used in Varian (1982) and Manser and McDonald (1988). However, this sort of improvement to the welfare bounds from revealed preference is only possible when budget surfaces cross. And this may, for the reasons discussed above, be a rare occurrence, particularly with aggregate or average demand data; indeed in Varian's (1982) applied work on GARP bounds on cost-of-living indices, improvements were only possible to the classical bounds for two years out of thirty two studied. In practise this has limited the usefulness of GARP bounds.

Knowledge of expansion paths can improve these bounds. Movements along expansion paths allow the maximum information on the curvature of the indifference curve through a given point to be utilised. The following algorithms provide upper and lower bounds on an indifference curve through a given point in commodity space:

[10] Under some circumstances a single adjustment designed to address one particular rejection may cause rejections elsewhere. Analogously it can also fix rejections elsewhere. There is, as far as we know, no easy way to tell which will happen in advance.

$RP(\mathbf{q}_0)$ **Bound Algorithm**

Output is the set RP of boundary points of which $\mathbf{q}_0$ is a member and which has $T+1$ elements where $\mathbf{p}_i'\mathbf{q}_i \leq \mathbf{p}_i'\mathbf{q}_j \ \forall \ \mathbf{q}_i, \mathbf{q}_j \in RP$ and either $\mathbf{q}_i \ R^0 \ \mathbf{q}_0$ or $\mathbf{q}_i \ R \ \mathbf{q}_0$ for all $\mathbf{q}_i \in RP$.

1) Set $W = \{\mathbf{q}_0\}$, $\tau = \{0, 1, ..., T\}$, $E = \varnothing$

2) Set $R = \{\mathbf{q}_t = (\min\{x \,|\mathbf{p}_t'\mathbf{q}_t = \mathbf{p}_t'\mathbf{q}_w\}) \ \mathbf{q}_w \in W, \ t \in \tau\}$

3) Set $E = \{\mathbf{q}_i \in R : \mathbf{p}_i'\mathbf{q}_i > \mathbf{p}_i'\mathbf{q}_j$ for $\mathbf{q}_j \in R\}$

4) Set $W = R/E$

5) If $E = \varnothing$ set $RP = W$ and stop. Otherwise go to (2).

$RW(\mathbf{q}_0)$ **Bound Algorithm**

Output is the set RW of boundary points of which $\mathbf{q}_0$ is a member and which has $T+1$ elements where $\mathbf{p}_i'\mathbf{q}_i \leq \mathbf{p}_i'\mathbf{q}_j \ \forall \ \mathbf{q}_i, \mathbf{q}_j \in RW$ and either $\mathbf{q}_0 \ R^0 \ \mathbf{q}_i$ or $\mathbf{q}_0 \ R \ \mathbf{q}_i$ for all $\mathbf{q}_i \in RW$.

1) Set $B = \{\mathbf{q}_0\}$, $\tau = \{0, 1, ..., T\}$, $E = \varnothing$

2) Set $R = \{\mathbf{q}_t = (\max\{x \,|\mathbf{p}_b'\mathbf{q}_b = \mathbf{p}_b'\mathbf{q}_t\}) \ \mathbf{q}_b \in B, \ t \in \tau\}$

3) Set $E = \left\{\mathbf{q}_i \in R : \mathbf{p}_j'\mathbf{q}_j > \mathbf{p}_j'\mathbf{q}_i \text{ for } \mathbf{q}_j \in R\right\}$

4) Set $B = R/E$

5) If $E = \varnothing$ set $RW = B$ and stop. Otherwise go to (2).

Proposition 4. *If the data local to the reference bundle* $\mathbf{q}_0$ *reject GARP, then the algorithm for the boundary to the set* $RP(\mathbf{q}_0)$ *will not converge.*

Proof.

Without any loss in generality take the simplest case. In which there are two periods. Denote the reference period bundle as $\mathbf{q}_0$, the other as $\mathbf{q}_1$.

1) By Step (1) $W = \mathbf{q}_0$, $t = \{0, 1\}, E = \varnothing$.

2) By Step (2) set x_1 such that $\mathbf{p}_1'\mathbf{q}_1(x_1) = \mathbf{p}_1'\mathbf{q}_0$. Set $R = \{\mathbf{q}_0, \mathbf{q}_1(x_1)\}$

3) Suppose that these data reject GARP. Since by construction $\mathbf{q}_1 R^0 \mathbf{q}_0$ this means that $\mathbf{q}_0 P^0 \mathbf{q}_1$.

4) By Step (3) $E = \{\mathbf{q}_0\}$ since $\mathbf{q}_0 P^0 \mathbf{q}_1$ through violation of GARP and by Step (4) $W = \{\mathbf{q}_1\}$.

5) Since $E \neq \varnothing$, return to Step (2) and set x_0' such that $\mathbf{p}_0'\mathbf{q}_0(x_0') = \mathbf{p}_0'\mathbf{q}_1$. Set $R = \{\mathbf{q}_0(x_0'), \mathbf{q}_1(x_1)\}$

6) Now by construction $\mathbf{q}_0(x_0')\,R^0\mathbf{q}_1$ but since $\mathbf{q}_0P^0\mathbf{q}_0(x_0')$ then (2) implies that $\mathbf{q}_1(x_1)\,P^0\mathbf{q}_0(x_0')$ which is a violation of GARP.

8) Hence by Step (3) $E = \{\mathbf{q}_1(x_1)\}$ through violation of GARP, and by Step (4) $W = \{\mathbf{q}_0(x_0')\}$.

9) Since $E \neq \varnothing$. Return to Step (2). etc. ■

Proposition 5. *If the data local to the reference bundle* $\mathbf{q}_0$ *reject GARP, then the algorithm for the boundary to the set* $RW(\mathbf{q}_0)$ *will not converge.*

Proof.

The proof is analogous with that for Proposition 4. ■

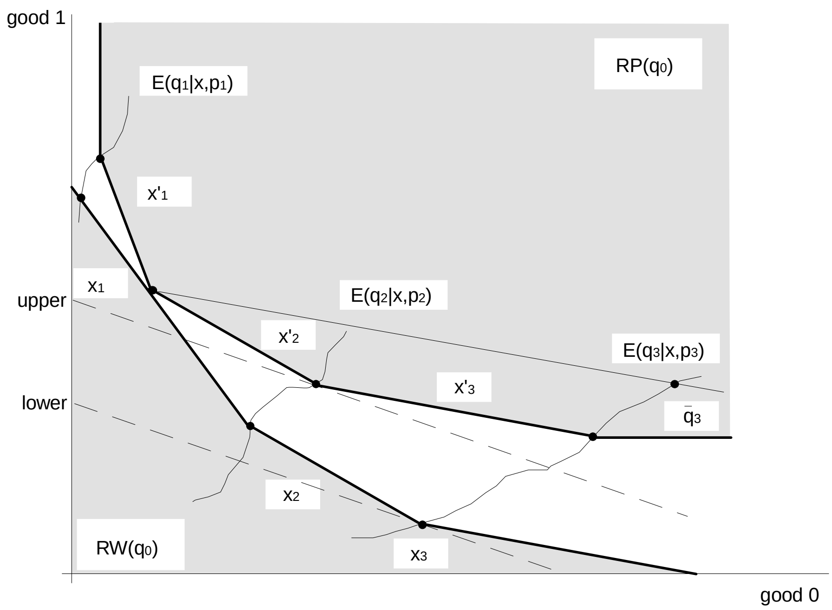
What this means is that these algorithms provide a test of GARP in the region around the reference bundle[11]. Indeed at each step bundles are found such that $\mathbf{p}_t'\mathbf{q}_t = \mathbf{p}_t'\mathbf{q}_0$. Using the argument for the SMP path described above, this maximises the possibility of finding the rejection $\mathbf{p}_0'\mathbf{q}_0 > \mathbf{p}_0'\mathbf{q}_t$, although the 'sequence' here is a set of pairwise comparisons either directly or transitively to $\mathbf{q}_0$. Propositions 3. and 4 tell us that if the algorithms fail to converge then there are no coherent indifference curves to bound in that region of the data because the data reject GARP.

Note that one of the benefits of this techniques over the classic bounds revealed by actual data in which any budget constraint can only appear once, is that each price regime can be used twice; once to bound the indifference curve from above, and once to bound it from below.

[11] The reason that the argument is restricted to commodity space local to the reference bundle is that expansion paths may cross and un-cross as they move through higher levels of total expenditure. Thus GARP may be rejected for, say high income households, but pass for low income households. The convergence of the algorithms requires that GARP is not violated in the region around the reference bundle.

A simple two dimensional illustration of the algorithms to compute the improved bounds is shown in figure 2.3. In this example budgets are set such that $\mathbf{p}_1'\mathbf{q}_1(x_1') = \mathbf{p}_1'\mathbf{q}_0,\ \mathbf{p}_2'\mathbf{q}_2(x_2') = \mathbf{p}_2'\mathbf{q}_0$, and hence $\mathbf{q}_1$ and $\mathbf{q}_2$ are directly preferred to $\mathbf{q}_0$ and are added to W in the first iteration. The point $\overline{\mathbf{q}_3}$ would also be identified in the first iteration since $\mathbf{p}_3'\overline{\mathbf{q}_3} = \mathbf{p}_3'\mathbf{q}_0$ but $\mathbf{p}_3'\overline{\mathbf{q}_3} > \mathbf{p}_3'\mathbf{q}_1$ and $\mathbf{p}_3'\overline{\mathbf{q}_3} > \mathbf{p}_3'\mathbf{q}_2$ imply $\overline{\mathbf{q}_3}\ P\ \mathbf{q}_0$ so $\overline{\mathbf{q}_3}$ must be above the indifference curve and so can be improved. In the next iteration we compute points revealed preferred to each of the (now three) members of $W = \{\mathbf{q}_0, \mathbf{q}_1(x_1'), \mathbf{q}_2(x_2')\}$. The points revealed preferred to $\mathbf{q}_0$ will just be $\{\mathbf{q}_0, \mathbf{q}_1(x_1'), \mathbf{q}_2(x_1'), \overline{\mathbf{q}_3}\}$ again.

Figure 2.3: Improving the bounds by means of expansion paths



There will be another four points (one on each expansion path including $E(\mathbf{q}_0|x, \mathbf{p}_0)$ which is not shown) which are revealed preferred to each of the other members of $W \;\; (\mathbf{q}_1 (x_1'), \mathbf{q}_2 (x_1'))$. These are placed in R, replacing the previous set and any strictly preferred bundles are removed to E. This leaves the $\mathbf{q}_3 (x_3')$ bundle at an expenditure level (x_3') such that $\mathbf{p}_3'\mathbf{q}_3 (x_3') = \mathbf{p}_3'\mathbf{q}_2 (x_2')$. Now $\mathbf{q}_3 (x_3') \;\; R^0 \;\; \mathbf{q}_2 (x_2')$ and $\mathbf{q}_2 (x_2') \;\; R^0 \;\; \mathbf{q}_0$ give $\mathbf{q}_3 (x_3') \;\; R \; \mathbf{q}_0$ and the algorithm ends with the upper bound illustrated as the next iteration will find no improvements, R will be identical to W and E will be an empty set. The budget lines using each price vector as the final total expenditure levels are denoted $\{x_1', x_2', x_3'\}$.[12]

3. Preference Heterogeneity

We turn now to the relationship between the (nonparametric) Engel curves above and the average demands for a set of heterogeneous agents. There are two alternative ways of interpreting the impact of heterogeneity on the average demands estimated from Kernel Engel curve regression. We could assume individual demands are rational and then ask for conditions on preferences and/or heterogeneity that imply rationality for average demands. This is the approach of McElroy (1987), Brown and Walker (1991) and Lewbel (1996). Alternatively, we could make no rationality assumptions on individual demands and simply ask what conditions enable average demands to satisfy rationality properties. This is the approach of Hildenbrand (1994) and Grandmont (1992).

Suppose for each good j we write average budget shares as

$$E\{w_j | \ln x, \mathbf{p}\} = f_j (\ln x, \mathbf{p}) \tag{3.1}$$

then, if we let $\boldsymbol{\varepsilon}$ represent the vector of unobserved heterogeneity terms, a nec-

[12] Note that the data illustrated give a good deal of information about the curvature of the region of indifference and the bounds in the welfare effects are tightened as a result since we can now discard the Paasche upper bound for the new price vector given by the dashed line.

essary condition for the average budget shares recovered by the nonparametric analysis discussed above to be equal to average budget shares is that:

$$w_j = f_j\left(\ln x, \mathbf{p}\right) + \phi_j\left(\ln x, \mathbf{p}\right)' \boldsymbol{\varepsilon} \tag{3.2}$$

where $E\left(\boldsymbol{\varepsilon}\right| \ln \mathbf{x}, \mathbf{p}) = 0$. Given this combination of functional form restrictions and distributional assumptions, our nonparametric analysis recovers $g_j^t(x) = f_j\left(\ln x, \mathbf{p}^t\right)$. Notice this allows for quite different tastes across agents. In particular, the first-order price and income responses for agents can vary in any way. Thus a good may be a luxury for one person and a neccessity for another.

This aggregation structure is very different to those used in Gorman (1954) and Muellbauer (1976). In particular, we are not aggregating across different incomes. Additionally, we are not assuming that individual demands are integrable; that is, for given $\boldsymbol{\varepsilon}$ we can have that the Slutsky conditions may fail for $w_j\left(\ln x, \mathbf{p}, \boldsymbol{\varepsilon}\right)$. In this respect, our structure is closer to that of Hildenbrand (1994) and Grandmont (1992). However, their analysis shows conditions for average demands to satisfy the Weak Axiom of Revealed Preference (WARP, see Varian (1982)) but GARP requires more. In particular, GARP implies the Slutsky symmetry conditions. In the heterogeneity structure given in (3.2) above we do not impose that individual demands satisfy the Slutsky conditions. If, however, we wish to impose integrability at the individual level then there are restrictions on the $\phi_j\left(x, \mathbf{p}\right)$ and the distribution of the heterogeneity terms (see McElroy (1987) and Brown and Walker (1989)). If all preference parameters are to be heterogeneous then preferences are restricted to what is essentially the class of Piglog demands (see Lewbel (1996), for example).

The function $f_j\left(\ln x, \mathbf{p}\right)$ gives mean responses to changes in prices conditional on a given level of total expenditure. Thus we can use this function for positive analysis, for example to recover the revenue implications from a change in taxes. Additionally, the utility function that is associated with an integrable set of demands $f_j(\ln x, \mathbf{p})$ is a prime candidate for use in equilibrium models that assume a representative agent. In our analysis below we apply the GARP tests to the mean function $f_j(\ln x, \mathbf{p})$. The reason that we are interested in testing for GARP using these mean responses is that without such a rationality condition holding, it is difficult to see how we would ever conduct coherent welfare analysis of price changes. The heterogeneity conditions for using the mean function for welfare analysis are, however, stronger than the conditions given in (3.2) which suffice for positive analysis.

To understand the biases that derive from using $f_j(\ln x, \mathbf{p})$ to conduct welfare analysis consider second order approximation of the log cost function[13] for a non-marginal price change $\Delta \ln p_j$.

$$\frac{\Delta \ln c}{\Delta \ln p_j} = \omega_j + \frac{1}{2}\left(\frac{\partial w_j}{\partial \ln p_j} + \frac{\partial w_j}{\partial \ln x} w_j\right) \Delta \ln p_j. \tag{3.3}$$

Using (3.2) this becomes

$$\frac{\Delta \ln c}{\Delta \ln p_j} = \omega_j + \frac{1}{2}\left[\left(\frac{\partial f_j}{\partial \ln p_j} + \frac{\partial f_j}{\partial \ln x} f_j\right) + \left(\frac{\partial \phi_j'}{\partial \ln p_j} + \frac{\partial \phi_j'}{\partial \ln x}\right) \boldsymbol{\varepsilon}(f_j + \phi_j' \boldsymbol{\varepsilon})\right] \Delta \ln p_j. \tag{3.4}$$

Therefore the mean welfare measure has the form

$$E\left[\frac{\Delta \ln c}{\Delta \ln p_j} | x, p\right] = \omega_j + \frac{1}{2}\left(\frac{\partial f_j}{\partial \ln p_j} + \frac{\partial f_j}{\partial \ln x} f_j\right) \Delta \ln p_j + \frac{1}{2} \frac{\partial \phi_j'}{\partial \ln x} \Omega_\varepsilon \phi_j \Delta \ln p_j. \tag{3.5}$$

[13] See Banks, Blundell and Lewbel (1996), for example.

where $E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'|x,\, p\} = \Omega_\varepsilon$.

The first two terms on the right hand side of this expression can be computed using the mean function $f_j\,(.)$ so that our mean function gives mean welfare effects if the final bias term is zero. This will be the case if, for example, the heterogeneity term $\phi(\ln x, \mathbf{p})$ is independent of total expenditure so that all households have the same marginal income effects. Note, however, that this condition is sufficient and not necessary; weaker assumptions suffice to make the bias term zero or small.

To illustrate, suppose that each household's preferences are Piglog. This covers the class of Almost Ideal (see Deaton and Muellbauer (1980)) and Translog (see Jorgenson, Lau and Stoker (1982) demand systems. The budget share for good j can be expressed as

$$w_j = \alpha_j + \Gamma_j(\mathbf{p}) + \beta_j\,(\ln x - \boldsymbol{\alpha}' \ln \mathbf{p} - \Gamma(\mathbf{p})' \ln \mathbf{p}) \text{ for } j = 1, .., n \tag{3.6}$$

where α_j and β_j are preference parameters $\Gamma(\mathbf{p})$ is a nonstochastic matrix of functions of prices (of which $\Gamma_j(\mathbf{p})$ is the j'th row) and $\boldsymbol{\alpha} = (\alpha_1, ...\alpha_n)'$. Allowing α_j and β_j to have additive random components v_j and η_j respectively results in a share model where the residual term is given by:

$$u_j = v_j - \beta_j \mathbf{v}' \ln \mathbf{p} + \eta_j \left(\ln x - (\boldsymbol{\alpha} + \mathbf{v})' \ln \mathbf{p} - \boldsymbol{\Gamma}\,(\mathbf{p})' \ln \mathbf{p}\right) \tag{3.7}$$

If we assume $E(\mathbf{v}|\ln x, \ln \mathbf{p}) = \mathbf{0}$, $E(\boldsymbol{\eta}|\ln x, \ln \mathbf{p}) = \mathbf{0}$ and $E(\mathbf{v}\boldsymbol{\eta}|\ln x, \ln \mathbf{p}) = \mathbf{0}$ we have the heterogeneity structure given in (3.2). However, notice also that if the heterogeneity is restricted to the $\mathbf{v}$ terms then there are no $\ln x$ terms in the heterogeneity expression (3.7) and the bias term disappears.

An alternative structure is suggested in Heckman (1974) and Brown and Matzkin (1995). In both of these papers the heterogeneity is introduced so that it enters in to the first order conditions in a convenient way. Specifically, these authors allow for multiplicative and additive heterogeneity terms, respectively, on the marginal utility of each good[14]. For example, Brown and Matzkin have:

$$\upsilon(\mathbf{q}, \boldsymbol{\varepsilon}) = \Psi(\mathbf{q}) + \mathbf{q}'\boldsymbol{\varepsilon}$$

where $\Psi(.)$ is the common utility function. This effectively makes the price heterogeneous since we have:

$$\lambda p_j = \upsilon_j(\mathbf{q}, \boldsymbol{\varepsilon}) = \Psi_j(\mathbf{q}) + \varepsilon_j$$

If heterogeneity of this form is introduced in to the simple AI model above it will be seen that the resulting error term does *not* fall in the structure given in (3.2). Thus we could not estimate consistently the average of budget shares using nonparametric techniques. Brown and Matzkin suggest an alternative strategy.

In general the error term in (3.2) will represent measurement and optimisation error as well as preference heterogeneity so it would seem natural to work with local average demands. Averaging locally to each x eliminates unobserved heterogeneity, measurement error and (zero mean) optimisation errors in demands but preserves any nonlinearities in the Engel curve relationship for each price regime.

4. An Empirical Investigation on Repeated Cross-Sections

4.1. Data

We use repeated cross-sections of household-level data from the British Family Expenditure Survey (1974 to 1993). The FES is a random sample of around

[14] Heckman (1974) actually adds a heterogeneity term to the marginal rate of substitution but this can be modelled as given in the text.

7,000 households per year from which a sub-sample of all the two-adult households with a car was drawn[15]. The first and last percentiles of the within-year total expenditure distribution in this sub-sample was then trimmed out. This leaves 58,947 households (between 2,640 and 3,242 in each year). Expenditures on non-durable goods by these households were aggregated into 22 commodity groups and chained Laspeyres price indices for these groups were calculated from the sub-indices of the UK Retail Price Index giving 20 annual price points for each group of goods.

The commodity groups are non-durable expenditures grouped into: beer, wine, spirits, tobacco, meat, dairy, vegetables, bread, other foods, food consumed outside the home, electricity, gas, adult clothing, children's clothing and footwear, household services, personal goods and services, leisure goods, entertainment, leisure services, fares, motoring and petrol. More precise definitions and descriptive statistics are provided Tables A.1 and A.2 in the Data Appendix.

4.2. Semiparametric Estimation and Observed Heterogeneity

A popular approach to semiparametric estimation is to use the following partially linear budget share regression (we drop the j subscripts denoting goods for convenience)

$$w = g(\ln x) + \mathbf{z}'\boldsymbol{\gamma} + \varepsilon \tag{4.1}$$

in which $\mathbf{z}'\boldsymbol{\gamma}$ represents a linear index in terms of a finite vector of observable exogenous regressors $\mathbf{z}$ and unknown parameter vector $\boldsymbol{\gamma}$. Here we may assume $E(\varepsilon|\mathbf{z}, \ln x) = 0$ and $Var(\varepsilon|\mathbf{z}, \ln x) = \sigma^2(\mathbf{z}, \ln x)$. Following Robinson (1988), a

[15] This was in order to allow us to include motoring and particularly petrol as commodity groups.

simple transformation of the model can be used to give an estimator for $\boldsymbol{\gamma}$. Taking expectations of (4.1) conditional on $\ln x$, and subtracting from (4.1) yields

$$w - E(w|\ln x) = (\mathbf{z} - E(\mathbf{z}|\ln x))'\boldsymbol{\gamma} + \varepsilon. \tag{4.2}$$

Replacing $E(w|\ln x)$ and $E(\mathbf{z}|\ln x)$ by their nonparametric estimators, denoted $\widehat{m}_h^w(\ln x)$ and $\widehat{\mathbf{m}}_h^z(\ln x)$ respectively, the ordinary least squares estimator for $\boldsymbol{\gamma}$ is $\sqrt{n}$ consistent and asymptotically normal.

The estimator for $g(\ln x)$ is then simply

$$\widehat{g}_h(\ln x) = \widehat{m}_h^w(\ln x) - \widehat{\mathbf{m}}_h^z(\ln x)'\widehat{\boldsymbol{\gamma}}. \tag{4.3}$$

Since $\widehat{\boldsymbol{\gamma}}$ converges at $\sqrt{n}$ the asymptotic distribution results for $\widehat{g}_h(\ln x)$ remain unaffected by estimation of $\boldsymbol{\gamma}$ and follows from the distribution of $\widehat{m}_h^w(\ln x) - \widehat{\mathbf{m}}_h^z(\ln x)'\boldsymbol{\gamma}$.

Although the partially linear model in (4.1) looks attractive, in this setting it turns out that imposing the integrability conditions requires that $g\,(.)$ be linear. To see this consider the Slutsky symmetry condition in budget share form:

$$\frac{\partial w_i}{\partial \ln p_j} + w_j \frac{\partial w_i}{\partial \ln x} = \frac{\partial w_j}{\partial \ln p_i} + w_i \frac{\partial w_j}{\partial \ln x} \tag{4.4}$$

Now suppose that budget shares have a form that is additive in functions of $\ln x$ and demogrphics, just as in (4.1):

$$w_i\,(\ln \mathbf{p}, \ln x, \mathbf{z}) = h^i\,(\ln \mathbf{p}, \mathbf{z}) + g^i\,(\ln \mathbf{p}, \ln x) \tag{4.5}$$

We then have the following proposition[16].

[16] In what follows let $\mathbf{g}_z^i$, for example, denote the vector of partial derivatives og $g^i\,(.)$ with respect to the vector of demographic $\mathbf{z}$.

Proposition 6. *If the budget shares take the form (4.5) and (i) Slutsky symmetry (4.4) holds (ii) the effects of demographics on budget shares are unrestrited in the sense that g_z^i can take any value, then $g^i(.)$ is linear in $\ln x$:*

$$g^i(\ln \mathbf{p}, \ln x) = \widetilde{g}^i(\ln \mathbf{p}) + \widehat{g}^i(\ln \mathbf{p}) \ln x.$$

Proof. Applying (4.4) to (4.5) we have

$$h_j^i + g_j^i + \left(h^j + g^j\right) g_x^i = h_i^j + g_i^j + \left(h^i + g^i\right) g_x^j.$$

Taking derivatives of both sides with respect to $\ln x$ then with respect to $\mathbf{z}$ gives:

$$\mathbf{h}_z^j g_{xx}^i = \mathbf{h}_z^i g_{xx}^j.$$

Invoking condition *(ii)* in the statement of the proposition we can set $\mathbf{h}_z^i$ equal to zero and $\mathbf{h}_z^j$ non-zero, which implies that $g_{xx}^i = 0$ so that $h^i(\ln \mathbf{p}, \ln x)$ is linear in $\ln x$. ■

This demonstartes that the additive form given in (4.1) will only be consistent with utility maximisation if we restrict the way in which demographics affect budget shares, or if preferences are Piglog.

There is, however, a similar form that is flexible and also consisitent with utility maximisation. This is the translated additive form for within-period responses:

$$w_{i0} = \alpha_i(\mathbf{z}) + g_i(\ln x - \phi(\mathbf{z})) \tag{4.6}$$

Budget shares (4.6) are a generalisation of the partially linear model. Interestingly this is precisely the shape invariance extension to the partially linear model considered in the work on pooling nonparametric regression curves in Hardle and Marron (1990) and Pinske and Robinson (1995). This analysis has recently been applied to the estimation of equivalence scales in the papers by Pendakur (1997) and Blundell, Duncan and Pendakur (1988). They note that independence of base equivalence scales results in (4.6) with log equivalence scales given by $\phi(z)$.

Suppose z is discrete, and $\phi(z) = \phi z$ and $\alpha_i(z) = \alpha_i z$ with a base group defined at z equal to zero. Suppose also that the unrestricted nonparametric regression has been estimated for each subgroup separately and write the resulting kernel regression estimates as

$$\widehat{g}_i^z = \frac{\widehat{r}_i^z}{\widehat{f}_i^z}. \tag{4.7}$$

The restrictions for the extended partially linear model (4.6) may be written

$$g_i^z = \alpha_i + g_i^z(\ln x - \phi z), \tag{4.8}$$

or

$$r_i^1 f_i^0 = f_i^1 r_i^0(\ln x - \phi z) + \alpha_i f_i^1 f_i^0(\ln x - \phi z). \tag{4.9}$$

Pinske and Robinson (1995) establish asymptotic convergence results for the estimator of α_i and ϕ that results from minimising the integrated squared loss function

$$L_N(\alpha_i; \phi) = \int \Lambda_N^2(\ln x; \alpha_i, \phi) w(\ln x) d \ln x \tag{4.10}$$

where

$$\Lambda_N(\ln x; \alpha_i, \phi) = r_i^1 f_i^0 - f_i^1 r_i^0(\ln x - \phi) - \alpha_i f_i^1 f_i^0(\ln x - \phi). \tag{4.11}$$

4.3. Endogeneity in log expenditure

To adjust for endogeneity we adapt the popular augmented regression technique (see Holly and Sargan (1982), for example) to the semiparametric framework. In particular, suppose x is endogenous in the sense that

$$E(\varepsilon | \ln x) \neq 0 \text{ or } E(w | \ln x) \neq g(\ln x). \tag{4.12}$$

In this case the nonparametric estimator will not be consistent for the function of interest. It will not provide the appropriate counterfactual: how do expenditure share patterns change for some given change in total expenditure? However, suppose there exists a variable ζ such that

$$\ln x = \pi\zeta + v \text{ with } E(v|\zeta) = 0. \tag{4.13}$$

Moreover, assume the following linear conditional model holds

$$w = g(\ln x) + v\rho + \varepsilon \tag{4.14}$$

with

$$E(\varepsilon | \ln x) = 0. \tag{4.15}$$

Note that

$$w - E(w | \ln x) = (v - E(v | \ln x))\rho + \varepsilon. \tag{4.16}$$

The estimator of $g(\ln x)$ is given by

$$\widehat{g}_h(\ln x) = \widehat{m}_h^w(\ln x) - \widehat{m}_h^v(\ln x)\widehat{\rho}. \tag{4.17}$$

In place of the unobservable error component v we use the first stage residuals

$$\widehat{v} = \ln x - \zeta\widehat{\pi} \tag{4.18}$$

where $\widehat{\pi}$ is the least squares estimator of π. Since $\widehat{\pi}$ and $\widehat{\rho}$ converge at $\sqrt{n}$ the asymptotic distribution for $\widehat{g}_h(\ln x)$ follows the distribution of $\widehat{m}_h^w(\ln x) -$

$\widehat{m}_h^v(\ln x)\rho$. Moreover, a test of the exogeneity null $H_0 : \rho = 0$, can be constructed from this least squares regression.

Newey, Powell and Vella (1995) have developed a generalisation of this idea for triangular simultaneous equation systems of the type considered here. They adopt a series approach to the estimation of the regression of w on $\ln x$ and v. This generalises the form of (4.14) and allows an assessment of the additive structure. They also use a nonparametric regression for the reduced form in place of the linear model (4.13). In our application we consider extending the model along these lines by including higher order terms in the residuals v and then testing the partially linear specification (4.14) against this more general additive recursive alternative (see also Blundell and Duncan (1998)). The first-stage residual $\hat{v}$ in is calculated using the log of disposable in come is used as the excluded instrumental variable.

4.4. Estimated Engel Curves and Normality

The three figures (4.1) to (4.3) show the estimated Engel curves (budget share against log total nominal expenditure) for 3 of our 22 commodities, for 3 of our 20 periods (1975 (circles), 1980 (squares), 1985 (triangles)). These represent a typical necessity (bread), a luxury (entertainment) and beer which displays a roughly quadratic logarithmic Engel curve behaviour.

On each Engel curve we plot the points on the chronological SMP paths which correspond to the 1st, 10th, 25th, 50th, 75th, 90th and 99th percentile points in the base year (1974). Pointwise 95% confidence bands at these points are also drawn. Note that, as we would expect, the precision is much lower at the tails of the outlay distribution. The left to right drift of the Engel curves apparent in these figure illustrates the growth in nominal expenditure which took place between these periods.

Figure 4.1: The Engel curve for Bread

Figure 4.2: The Engel curve for Entertainment

Normality of demands was necessary for the proof of the properties of the SMP path which we intend to exploit in our test of GARP. Non-parametric regressions

Figure 4.3: The Engel curve for Beer

of quantity demanded for each commodity for 3 years of data against log total spending are presented in Appendix A for 1975, 1980 and 1985. There are no instances of goods behaving as inferior goods. The years illustrated are typical. The rest of the expansion paths for the remaining periods are available from the authors.

4.5. Results

Testing GARP

We proceed by estimating non-parametric Engel curves for each commodity group within each time period and calculate $g_t^j(\ln x_t)$ at various comparison points in the total expenditure distribution. We also control for the number of children in the household as described in section 4.2 above. In the first year of our data we selected the comparison points to be at the 1st percentile, 1st decile, 1st quartile, median, 3rd quartile, 9th decile and 99th percentile points. The comparison points for the following years were chosen to maximise the power of the test on a chronological SMP path which orders the data according to $\mathbf{q}_{t+1}\ R^0\ \mathbf{q}_t\ R^0\ \mathbf{q}_{t-1}$ as described above. By Proposition 1 we know that if this path passes GARP then no path which preserves the same preference ordering will violate GARP. We also present the annual median and mean (non-SMP) paths for comparison. We define a $(T \times T)$ indicator matrix m and compute the transitive closure $\widetilde{m}$ in which we know that by construction of the SMP path every element in the lower triangle must be one since either $\mathbf{q}_t\ R\ \mathbf{q}_{t-i}$ or $\mathbf{q}_t\ P\ \mathbf{q}_{t-i}$. We then check for rejections in the corresponding direct and transitive comparisons i.e. if $\mathbf{q}_t\ R\ \mathbf{q}_{t-i}$ (or $\mathbf{q}_t\ P\ \mathbf{q}_{t-i}$)in the lower triangle then $\mathbf{q}_{t-i}\ P\ \mathbf{q}_t$ (or $\mathbf{q}_{t-i}\ R\ \mathbf{q}_t$) in the upper triangle indicates a rejection of GARP.

Table 4.1: Number of rejections of GARP, by size of test.

Comparison paths	**Size of test**					
	Raw	**0.300**	**0.200**	**0.100**	**0.050**	**0.010**
SMP path starting points:						
1st percentile point	1	1	1	0	0	0
1st decile point	1	1	1	1	1	0
1st quartile point	1	1	0	0	0	0
Median	1	1	1	1	1	1
3rd quartile point	2	2	2	0	0	0
9th decile point	11	6	6	3	1	1
99th percentile point	28	21	21	1	0	0
Median	0	0	0	0	0	0
Mean	0	0	0	0	0	0

Table 4.1 shows the number and pattern of rejections for the our full budget system of 22 goods. Each column refers to a different size of test at each point.

As can be seen, GARP is rejected for a large number of points in the upper tail of the outlay distribution but these rejections are not very 'significant' statistically. GARP is, however, rejected in the data at the 1% level for the median SMP path and the SMP path starting at the 9th decile point. However, in each case, the rejection only occurs for a single comparison point and, except for this point, the large number of rejections in the raw data are considerably reduced by the use of pointwise confidence bands.

It is interesting to observe that there are no rejections even in the raw data for the median or mean (non-SMP) paths. This is consistent with the observation which arises in tests of GARP on aggregate data that if the budget constraint is allowed to shift much either way between comparison points, as it does for median or mean total expenditure, then there is little chance of being able to find demands that cannot be rationalised.

From the GARP test results we note that there are many rejections of revealed preference conditions in the upper tail of the total expenditure distribution and a few in the middle. As discussed above the algorithms we present which are designed to bound well-behaved indifference curves will only converge if there are well-behaved indifference curves to bound. Given the rejection of GARP in the raw data the algorithms will not be able to find coherent indifference curves using data for the entire period. However, we can run the algorithms for non-rejecting sub-periods and the results of this are presented in Table 4.2.

The table shows the largest continuous sub-period in which the algorithms are able to bound the indifference curve. For example, for the starting point at

Table 4.2: Continuous periods of convergence.

<table>
<tr><th rowspan="2"></th><th colspan="20">Periods</th></tr>
<tr><th>74</th><th>75</th><th>76</th><th>77</th><th>78</th><th>79</th><th>80</th><th>81</th><th>82</th><th>83</th><th>84</th><th>85</th><th>86</th><th>87</th><th>88</th><th>89</th><th>90</th><th>91</th><th>92</th><th>93</th></tr>
<tr><td>1st</td><td colspan="9"></td><td colspan="5"></td><td colspan="6"></td></tr>
<tr><td>10th</td><td colspan="14"></td><td colspan="6"></td></tr>
<tr><td>25th</td><td colspan="12"></td><td colspan="8"></td></tr>
<tr><td>50th</td><td colspan="12"></td><td colspan="8"></td></tr>
<tr><td>75th</td><td colspan="12"></td><td colspan="5"></td><td colspan="3"></td></tr>
<tr><td>90th</td><td colspan="12"></td><td colspan="5"></td><td colspan="3"></td></tr>
<tr><td>99th</td><td colspan="11"></td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td><td colspan="3"></td></tr>
</table>

median total outlay in 1974 we are able to bound a curve using the expansion paths and price data for 1974 to 1985 inclusive, and using any of the periods within the interval as the base. If we add 1986 to the set of admissible periods the algorithm fails to converge (we already know from the SMP path that 1985 and 1986 are not rationalisable). We then start again using the 1986 point on the median SMP path as our starting point. In all, for the median we find the entire period breaks down into two sub-periods within which we are able to bound an indifference curve. Similarly the 1st and 9th decile paths break into two and four sub-periods respectively, while the 99th percentile breaks down into five.

Allowing for Changes in Preferences

The GARP test itself gives us no clue as to the good or goods which are causing the rejection. We choose tobacco as our conditioning good and argue that we have reasonable prior belief that preferences for tobacco may have changed over the period with the arrival of new information on the health effects of smoking. We apply the conditioning procedure to the Median SMP path to the path of demands illustrated below. This allows us to account for changes preferences

Figure 4.4: The median SMP quantity path for tobacco.

by calculating an adjusted price for tobacco which is consistent with GARP and should, for example, allow the run of non-rejecting periods to be extended passed the first rejection on the median SMP path.

In this case the rejection is being caused by $\mathbf{p}'_{85}\mathbf{q}_{85} > \mathbf{p}'_{85}\mathbf{q}_{86}$ while the chronological SMP path requires $\mathbf{p}'_{86}\mathbf{q}_{86} = \mathbf{p}'_{86}\mathbf{q}_{85}$. Figure 4.4 shows the quantity of tobacco demanded on the median SMP expenditure path. Between 1985 and 1986 quantity demanded falls by 8.9%. At the same time there is a price increase of just 7.41%. The minimum neceassary adjustment of the price such that it rationalises the demand fall is such that the virtual price increases by 17.5% between 1985 and 1986.

Table 4.3 below report the results for the re-run of the GARP test with the actual price series for tobacco replaced by the virtual price in 1985. Of course this adjustment changes the relative prices not just between the years in question (85/86), but between 1985 and every other year. Further, given that the confidence intervals for the pairwise tests are based on price-weighted sums of kernel regressions, changes in the prices can affect the standard error of the price/demand bundles and changes the results of the statistical tests of GARP. Table 4.3 reports the new evaluation of GARP in which the price of tobacco in 1985 is replaced by its virtual price. Looking at the column of rejections of GARP in the raw data we see that this change has also corrected rejections on the 1st and 3rd quartile SMP paths. This is because these rejections also referred to the 85/86 comparisons and in each case the minimum virtual price adjustment to the tobacco price was less than that required by the median path. The adjustment for the 1st quartile SMP path was such that, in order to rationalise the 7.6% demand fall, a minimum price increase of 9.9% was required. The adjustment for the 3rd quartile SMP path was such that, in order to rationalise the 11.5% demand fall, a minimum price increase of 12% was required.

Table 4.3: Number of rejections of GARP, by size of test, virtual price of tobacco (1985).

Comparison paths	**Size of test**					
	Raw	**0.300**	**0.200**	**0.100**	**0.050**	**0.010**
SMP path starting points:						
1st percentile point	1	1	1	0	0	0
1st decile point	1	1	1	1	1	0
1st quartile point	0	0	0	0	0	0
Median	0	0	0	0	0	0
3rd quartile point	1	1	1	0	0	0
9th decile point	10	6	6	3	1	0
99th percentile point	28	15	15	1	0	0
Median	0	0	0	0	0	0
Mean	0	0	0	0	0	0

Bounds on the True Cost-of-living Index

Using the virtual price of tobacco for the median SMP path we derive GARP-based bounds on the cost-of-living index over the period 1974 to 1993 using welfare at 1974 median total expenditure as our reference welfare level. The results are reported in Tables 4.4 and 4.5. Table 4.4 summarises some popular parametric indices, some of which represent first-order, and some of which represent second-order approximations to true indices based on any arbitrary cost-function. These indices can also be thought of as corresponding exactly to true indices under various assumptions regarding the precise form of preferences (the Paasche and Laspeyres, for example, are exact for Leontief preferences, the Törnqvist is exact for translog).Table 4.5 reports various nonparametric bounds which have been suggested in the literature and the GARP-based bounds (in the final column) derived using the algorthims described above.

Table 4.5 reports the bounds on the true 1974 median welfare-based cost of living index which can be derive without *any* assumption on functional forms. The bounds provided by Lerner (1935-36) are simply that the true index (being a weighted average of price changes) must lie somewhere between the maximum and the minimum ratio of the price changes of all goods: i.e .

$$\min_i \left\{ \frac{p_t^i}{p_{74}^i} : i = 0, 1, ..., n \right\} \leq \frac{c(u_{74}, p_t)}{c(u_{74}, p_{74})} \leq \max_i \left\{ \frac{p_t^i}{p_{74}^i} : i = 0, 1, ..., n \right\}.$$

Pollak (1971) improves this by linking Lerner's result with the original Konüs (1924) result that the Laspeyres index approximates the true base-referenced cost of living index from above, i.e.

$$\frac{\mathbf{p}_t'\mathbf{q}_{74}}{\mathbf{p}_{74}'\mathbf{q}_{74}} \leq \frac{c(u_{74}, \mathbf{p}_t)}{c(u_{74}, \mathbf{p}_{74})} \leq \max_i \left\{ \frac{p_t^i}{p_{74}^i} : i = 0, 1, ..., n \right\}.$$

The bounds from classical revealed preference (GARP) restrictions of the type used by Varian (1982) (i.e. those which utilise the restrictions implied by prices and demands at observed annual mean expenditures to bound a level set of utility in commodity space) are reported in the next column and the bounds derived from the procedure we describe above are reported in final column.

Table 4.4: GARP bounds and popular price indices, 1974 to 1993; virtual price of tobacco (1985).

Year	Paasche	Laspeyres	Fisher's	Törnqvist	Törnqvist (Chained)	Divisia	Divisia (Chained)
74	1000	1000	1000	1000	1000	1000	1000
75	1215	1232	1223	1223	1220	1226	1223
76	1516	1530	1523	1523	1520	1529	1528
77	1762	1787	1775	1774	1774	1781	1783
78	1931	1957	1944	1944	1946	1950	1960
79	2086	2119	2102	2102	2106	2106	2121
80	2463	2514	2488	2489	2493	2494	2514
81	2780	2841	2810	2812	2814	2833	2841
82	3093	3189	3140	3143	3148	3168	3178
83	3260	3381	3320	3325	3337	3361	3371
84	3408	3558	3482	3488	3498	3527	3534
85	3541	3709	3624	3633	3646	3674	3685
86	3700	3911	3804	3812	3833	3848	3875
87	3825	4035	3929	3936	3946	3985	3990
88	3922	4163	4041	4052	4064	4103	4112
89	3413	4379	4253	4262	4270	4327	4321
90	4406	4669	4536	4550	4553	4623	4607
91	4723	5044	4881	4898	4907	4988	4966
92	4996	5437	5212	5236	5258	5347	5322
93	5177	5650	5409	5430	5428	5582	5498

Table 4.5: GARP bounds and other nonparametric bounds, 1974 to 1993; virtual price of tobacco (1985).

Year	Lerner	Pollak	Classical RP	GARP
74	1000	1000	1000	1000
75	[1025,1721]	[1025,1232]	[1206,1232]	[1215,1228]
76	[1182,1985]	[1182,1530]	[1431,1530]	[1515,1530]
77	[1239,2590]	[1239,1787]	[1700,1787]	[1763,1781]
78	[1385,2513]	[1385,1957]	[1894,1957]	[1937,1957]
79	[1461,2636]	[1461,2119]	[2058,2119]	[2094,2119]
80	[1734,3142]	[1734,2514]	[2442,2514]	[2479,2509]
81	[1770,4077]	[1770,2841]	[2687,2841]	[2802,2838]
82	[1821,4287]	[1821,3189]	[2983,3189]	[3124,3172]
83	[1828,4924]	[1828,3381]	[3197,3381]	[3316,3369]
84	[1790,4921]	[1790,3558]	[3329,3558]	[3474,3530]
85	[1836,5022]	[1836,3709]	[3228,3709]	[3622,3682]
86	[1900,5463]	[1900,3911]	[3308,3911]	[3809,3873]
87	[1920,6049]	[1920,4035]	[3300,4035]	[3919,3988]
88	[1923,6143]	[1923,4163]	[3370,4163]	[4037,4109]
89	[1996,6397]	[1996,4379]	[3356,4379]	[4241,4318]
90	[2079,6637]	[2079,4669]	[3403,4669]	[4521,4603]
91	[2109,7507]	[2109,5044]	[3911,5044]	[4870,4961]
92	[2091,8353]	[2091,5437]	[3888,5437]	[5212,5316]
93	[2066,9098]	[2066,5650]	[3841,5650]	[5379,5491]

We find, as did Varian (1982) and Manser and McDonald (1988) that classical non-parametric/revealed preference bounds based on the average demand data gives little additional information on the curvature of the indifference curve through commodity space and hence the bounds on the true index are wide. However, by the use of expansion paths we can dramatically improve these bounds. Although neither index is base period utility referenced[17], in practice both the

[17] The Törnqvist index which links period t with period s, for example, is referenced at the utility level $(u_s u_t)^{1/2}$.

Figure 4.5: GARP bounds, the Paasche and Laspeyres indices, 1974 to 1993.

Törnqvist index, Fisher's index and the Divisia perform well in approximating the true index. Figure 4.5 illustrates the Paasche and Laspeyres indices and the GARP bounds. The GARP bounds derived using the methods described are given by the dashed lines. Figure 4.6 illustrates the GARP bounds and the Lerner and Pollak nonparametric bounds. The Lerner bounds are the outer solid lines, the Pollak lower bound is the same and the Lerner lower bound, the upper bound is the lower of the two solid lines above the GARP bounds. This corresponds exactly to the Laspeyres index. Figure 4.7 illustrates the classical RP bounds of the type calculated by Varian (1982) and the GARP bounds.

Figures 4.8 to 4.11 show indices from tables 4.4 and 4.5 expressed as propro-tional differences from the centre of the GARP bounds. From these we can see that the GARP bounds represent approximately $\pm 2\%$ of the level of the centre of the bounds by 1993. The Paasche and Laspeyres indices under- and over-state

Figure 4.6: Lerner and Pollak bounds and GARP bounds, 1974 to 1993.

Figure 4.7: Classical RP bounds and GARP bounds, 1974 to 1993.

the true index by 5% and 4% of the level by 1993. Even though they are not u_{74}-referenced, the chained Divisia and the Törnqvist indices give good *de facto* approximations to the true index. The improvement to the previously available nonparametric bounds which our procedure gives is highlighted in figure 4.11 which shows that the lower classical RP bound capture only 70% of the increase in the true cost of living over the period.

Figure 4.8: GARP bounds, Paasche and Laspeyres indices; proportional difference from the centre of the GARP bounds, 1975 to 1993.

The cost of living indices so far presented were based on the utility level associated with median 1974 total outlay (the rejection between 1985 and 1986 was overcome by utilising the conditioning procedure described in section 2.5). We can begin to investigate the distributional effects of price changes by applying the same procedure to different points in the total outlay distribution. Figure 4.12 shows the GARP bounds on the true cost of living index which compares 1985

Figure 4.9: GARP bounds, chained and unchained Törnqvist indices; proportional difference from the centre of the GARP bounds, 1975 to 1993.

Figure 4.10: GARP bounds, chained and unchained Divisia indices; proportional difference from the centre of the GARP bounds, 1975 to 1993.

Figure 4.11: GARP and Classical RP bounds; proportional difference from the centre of the GARP bounds, 1975 to 1993.

and 1974, for the 10th and the 90th percentile points of the 1974 total outlay distribution (as table 4.2 showed, this is the longest period over which we can bound coherent indifference curve for both of these starting points without using virtual prices). The bounds for 10th and 90th percentile points do not overlap and indicate greater rise on the cost of living for poorer, compared to richer, households over this period period.

5. Summary and Conclusion

In this paper we have applied nonparametric demand theory to the nonparametric statistical analysis of consumer demand. We exploit the idea that price taking individuals in the same market face the same relative prices, in order to

Figure 4.12: GARP Cost of living index bounds 1985 by percentile point, 1974=1000.

smooth across the demands of individuals for each common price regime. We show how this provides a conventional stochastic structure within which to examine the consistency of individual data and revealed preference theory. We also presents a method of maximising the power of these tests of revealed preference. We have discussed the way in which we might allow for taste changes for a good from which the other goods are not separable. We also present algorithms which allow us to place bounds on level sets of utility in commodity space. This allows the calculations of improved bounds on true cost-of-living indices. The heterogeneity conditions for carrying out both positive and welfare analysis were also discussed.

Using a long time series of repeated cross-sections from the 1974-1993 British Family Expenditure Surveys we were able to examine whether revealed preference theory is rejected. We show that GARP is not rejected for long periods, particularly when we allow for sampling/stochastic variation. Allowing for taste changes in tobacco in one year is shown to reduce the number of the rejections further. We also show that we can derive bounds on cost of living indices from our analysis which appear to be much tighter than those based on the revealed preference restrictions implied by demands at, say, annual mean total expenditure. We also note that, (despite the fact that neither of these indices are themselves base period referenced) the Törnqvist and the chained Divisia indices perform well as empirical approximations to the true base-period referenced index.

Appendices

A. Data Appendix

The 22 commodity groups are defined as

Table A.1: Commodity definitions

Commodity Group	
Beer	Beer, on and off licence sales.
Wine	Wine, on and off licence sales.
Spirits	Spirits, on and off licence sales.
Meat	All meat & fish
Dairy	All diary products, oils and fats.
Vegetables	Fresh, tinned and dried vegetables & fruit.
Bread	Bread, flour, rice & cereals.
Other foods	Tea, coffee, drinks, sugar, jams & sweets.
Food consumed outside the home	Restaurant & canteen meals.
Electricity	Account & slot meter payments.
Gas	Account & slot meter payments.
Adult clothing	Adult clothing
Children's clothing and footwear	Children's clothing & footwear
Household services	Post, phone, domestic services & fees.
Personal goods and services	Personal & chemist's goods & services.
Leisure goods	Records, CD's, toys, books & gardening.
Entertainment	Entertainment.
Leisure services	TV licences & rentals.
Fares	Rail, bus & other fares.
Motoring	Maintenance, tax & insurance.
Petrol	Petrol & oil
Tobacco	Cigarettes, pipe tobacco & cigars.

Table A.2: Total nominal expenditure: Annual descriptive statistics.

Year	*No. of Obs*	*Mean*	*Std Dev.*	*10%*	*50%*	*90%*
1974	2640	35.11	15.55	19.00	31.97	54.20
1975	2828	41.92	18.18	22.80	38.45	64.60
1976	2780	47.09	20.51	25.61	42.95	73.21
1977	2854	54.35	24.31	29.94	49.54	85.24
1978	2751	59.59	25.17	32.96	54.92	90.97
1979	2698	70.23	31.61	37.29	64.01	109.54
1980	2885	82.94	36.75	44.60	76.17	128.61
1981	3111	90.62	41.57	49.53	82.03	142.41
1982	3001	96.60	42.61	52.78	87.94	151.36
1983	2898	105.69	49.05	56.54	95.71	166.93
1984	2863	111.03	53.75	58.33	99.00	175.81
1985	2915	118.60	57.53	60.49	106.58	190.94
1986	2918	126.42	61.63	64.44	111.96	206.23
1987	3060	136.23	68.35	67.74	120.94	225.42
1988	3152	147.86	76.44	71.10	130.72	241.23
1989	3242	157.44	78.98	78.29	140.64	255.78
1990	2968	174.51	92.07	84.65	154.41	286.29
1991	3088	183.12	94.27	90.52	162.94	300.85
1992	3217	188.37	88.08	95.35	170.39	301.20
1993	3078	202.64	107.44	99.37	177.57	333.83

B. Quantity against log Total Nominal Expenditure for each commodity group; 1975, 1980 and 1985.

These figures show adaptive kernel estimates of the relationship between quantity demanded for each good and the log of total nominal household expenditure on all goods for three years of the data; 1975, 1980 and 1985. Each kernel regression has the percentile (1st, 10th, 25th, 50th, 75, 90th, 99th) chronological SMP path points and corresponding pointwise 95% confidence intervals marked on the curve.

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